TRANSIENT PROCESSES IN A THERMAL DIFFUSION APPARATUS WITH TRANSVERSE FLOWS

A. V. Suvorov and G. D. Rabinovich

UDC 621.039.341.6

Transient processes in a thermal diffusion apparatus with transverse flows operating in both the open and closed cycles are considered. The results obtained are analyzed.

A theory of a thermal-diffusion apparatus with transverse flows was developed in [1], and theoretical relations are obtained for the steady state, enabling the region in which it is best to use such devices to be determined.

In this paper we consider transient processes in an apparatus the basic scheme of which is shown in Fig. 1. Region I of height L is the separating part of the apparatus, while in channels II and III, having a constant cross section x, the separated mixture moves, and this motion can be either unidirectional (forward flow, Fig. 1a), or in opposite directions (counter flow, Fig. 1b). In formulating this problem mathematically we will make the following assumptions [1]: 1) in the lower and upper channels in a vertical direction ideal mixing of the flows σ_e, σ_i occurs with the flows τ^* arriving from the separating part of the apparatus I; 2) diffusion along the x axis inside the separating part of the apparatus can be neglected; this condition is satisfied satisfactorily when there is a fairly large number of discrete columns; 3) we will neglect the longitudinal diffusion in channels II and III, which corresponds to the condition

$$\sigma\Delta c \gg f\rho D\frac{dc}{dx} \quad \text{or} \quad \sigma \gg \frac{\rho D f}{B}. \tag{1}$$

The transfer in the direction of the z axis per unit width of the column is given by

$$\tau^* = H^*\left[c(1-c) - \frac{\partial c}{\partial y}\right], \tag{2}$$

where

$$H^* = \frac{H}{B} = \frac{\alpha\rho^2 g\beta\delta^3(\Delta T)^2}{6!\ \eta T}; \quad y = \frac{HL}{K}.$$

For a transient process in a section of column of width dx we have

$$m^*\frac{\partial c}{\partial t} = -\frac{\partial \tau^*}{\partial z}. \tag{3}$$

We will take the following as the boundary conditions which solution (3) must satisfy:

$$(\tau^* dt = f_e\rho dc)_{y=y_e}, \quad (\tau^* dt = f_i\rho dc)_{y=0}, \tag{4}$$

which shows that the change in the amount of transferred component in the upper and lower parts of the separating part of the apparatus along a section dx after an infinitely small time is equal to the change in the concentration along this part in the upper and lower channels (in this case we have made use of assumption 1), and also the fact that the purposeful component is concentrated at the top of the column.

Conditions (4) can be rewritten as follows:

$$\left(\tau^* = f_e\rho\frac{dc}{dt}\right)_{y=y_e}, \quad \left(\tau^* = -f_i\rho\frac{dc}{dt}\right)_{y=0}. \tag{5}$$

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the B. SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 39, No. 1, pp. 86-95, July, 1980. Original article submitted June 15, 1979.

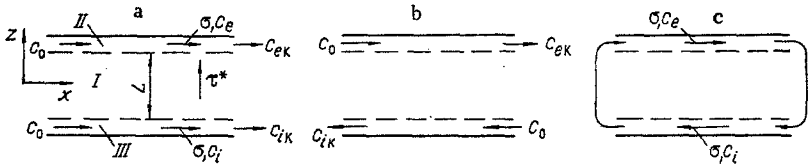

Fig. 1. Scheme showing the input and output of the separated mixture in the apparatus: a) forward flow; b) counter flow; c) closed scheme.

In (5) dc/dt is a substantial derivative, i.e., it represents the change in the concentration in time and space, and, taking into account the fact that the problem is one-dimensional

$$\frac{dc}{dt}=\frac{\partial c}{\partial t}+w\frac{\partial c}{\partial x}.$$

Consequently, conditions (5), taking (2) into account, have the form

$$H^*\left[c(1-c)-\frac{\partial c}{\partial y}\right]_{y=y_e}=\left[f_e\rho\frac{\partial c}{\partial t}+w_e\rho f_e\frac{\partial c}{\partial x}\right]_{y=y_e},$$
$$H^*\left[c(1-c)-\frac{\partial c}{\partial y}\right]_{y=0}=-\left[f_i\rho\frac{\partial c}{\partial t}+\rho f_i w_i\frac{\partial c}{\partial x}\right]_{y=0}. \tag{6}$$

We will introduce one more assumption regarding the smallness of the first terms on the right side of these equations compared with the second terms. This means that we can confine ourselves to considering times that satisfy the condition

$$t\gg\frac{B}{w}, \tag{7}$$

i.e., times which far exceed the time taken for the liquid to flow through the channel. In addition, we will assume, in order to simplify the last calculations, that the separated mixture satisfies the condition $c(1-c)=a$, i.e., a constant quantity, while $\sigma_e=\sigma_i=\sigma$. Then, after introducing the dimensionless variables

$$\omega=\frac{M}{mL},\quad \theta=\frac{H^2t}{mK},\quad \xi=\frac{x}{B},\quad \varkappa=\frac{\sigma}{H} \tag{8}$$

taking (3) and (6) into account, the problem reduces to solving the heat-conduction equation

$$\frac{\partial u}{\partial\theta}=\frac{\partial^2 u}{\partial y^2} \tag{9}$$

with the boundary condition

$$\left[\frac{\partial u}{\partial y}+\varkappa\frac{\partial u}{\partial\xi}\right]_{y=y_e}=a,\quad \left[\frac{\partial u}{\partial y}-\varkappa\frac{\partial u}{\partial\xi}\right]_{y=0}=a. \tag{10}$$

The second of conditions (10) is written for the case when the liquid in both channels moves in the same direction (forward flow). In the case of counter flow, due to the change in the direction of the flow of liquid in the lower channel, this condition takes the form

$$\left[\frac{\partial u}{\partial y}+\varkappa\frac{\partial u}{\partial\xi}\right]_{y=0}=a, \tag{11}$$

where

$$u=c-c_0. \tag{12}$$

We will take as the initial condition

$$u|_{\theta=0}=0. \tag{13}$$

In Laplace–Carson transforms the solution of (9) will be

$$\bar{u}=Ae^{\sqrt{p}\,y}+Be^{-\sqrt{p}\,y}, \tag{14}$$

where we must bear in mind that A and B are functions of the longitudinal coordinate ξ, i.e., $A=A(\xi)$, $B=B(\xi)$.

Forward Flow. Using conditions (10), we obtain the following two equations for $A(\xi)$ and $B(\xi)$ using (14):

$$\varkappa e^{\sqrt{p}y_e}A'(\xi)+\sqrt{p}\,e^{\sqrt{p}y_e}A(\xi)+\varkappa e^{-\sqrt{p}y_e}B'(\xi)-\sqrt{p}\,e^{-\sqrt{p}y_e}B(\xi)=a,$$
$$-\varkappa A'(\xi)+\sqrt{p}\,A(\xi)-\varkappa B'(\xi)-\sqrt{p}\,B(\xi)=a.$$

The solution of this system gives

$$A(\xi)=C_1\exp\left(-\frac{\sqrt{p}}{\varkappa}\xi\,\mathrm{th}\sqrt{p}\frac{y_e}{2}\right)+C_2\exp\left(-\frac{\sqrt{p}}{\varkappa}\xi\,\mathrm{cth}\sqrt{p}\frac{y_e}{2}\right)+\frac{a}{2\sqrt{p}}\left(1-\mathrm{th}\sqrt{p}\frac{y_e}{2}\right),$$
$$B(\xi)=C_1\frac{1+\mathrm{th}\sqrt{p}\frac{y_e}{2}}{1-\mathrm{th}\sqrt{p}\frac{y_e}{2}}\exp\left(-\frac{\sqrt{p}}{\varkappa}\xi\,\mathrm{th}\sqrt{p}\frac{y_e}{2}\right)-C_2\frac{1+\mathrm{th}\sqrt{p}\frac{y_e}{2}}{1-\mathrm{th}\sqrt{p}\frac{y_e}{2}}\exp\left(-\frac{\sqrt{p}}{\varkappa}\xi\,\mathrm{cth}\sqrt{p}\frac{y_e}{2}\right)$$
$$-\frac{a}{2\sqrt{p}}\left(1+\mathrm{th}\sqrt{p}\frac{y_e}{2}\right),$$

where C_1 and C_2 are constants independent of ξ.

Substituting the expressions obtained into Eq. (14) and assuming that $y = y_e$ and $y = 0$, we obtain the following expressions for representing the shift in the concentration in the upper and lower channels (the subscripts e and i):

$$\bar{u}_e=2C_1\frac{\exp\left(-\frac{\sqrt{p}}{\varkappa}\xi\,\mathrm{th}\sqrt{p}\frac{y_e}{2}\right)}{1-\mathrm{th}\sqrt{p}\frac{y_e}{2}}+2C_2\frac{\mathrm{th}\sqrt{p}\frac{y_e}{2}}{1-\mathrm{th}\sqrt{p}\frac{y_e}{2}}\exp\left(-\frac{\sqrt{p}}{\varkappa}\xi\,\mathrm{cth}\sqrt{p}\frac{y_e}{2}\right)+\frac{a}{\sqrt{p}}\mathrm{th}\sqrt{p}\frac{y_e}{2},\tag{15}$$

$$\bar{u}_i=2C_1\frac{\exp\left(-\frac{\sqrt{p}}{\varkappa}\xi\,\mathrm{th}\sqrt{p}\frac{y_e}{2}\right)}{1-\mathrm{th}\sqrt{p}\frac{y_e}{2}}-2C_2\frac{\mathrm{th}\sqrt{p}\frac{y_e}{2}}{1-\mathrm{th}\sqrt{p}\frac{y_e}{2}}\exp\left(-\frac{\sqrt{p}}{\varkappa}\xi\,\mathrm{cth}\sqrt{p}\frac{y_e}{2}\right)-\frac{a}{\sqrt{p}}\mathrm{th}\sqrt{p}\frac{y_e}{2}.\tag{16}$$

For forward flow the concentrations at the input of both channels are equal to the initial concentration c_0, which enables us to use the following conditions to obtain C_1 and C_2:

$$\bar{u}_e|_{\xi=0}=0,\ \bar{u}_i|_{\xi=0}=0.\tag{17}$$

From these conditions and also (15) and (16) we obtain

$$C_1=0,\ C_2=-\frac{a}{2\sqrt{p}}\left(1-\mathrm{th}\sqrt{p}\frac{y_e}{2}\right)$$

and finally

$$\bar{u}_e=\frac{a}{\sqrt{p}}\mathrm{th}\sqrt{p}\frac{y_e}{2}\left[1-\exp\left(-\frac{\sqrt{p}}{\varkappa}\xi\,\mathrm{cth}\sqrt{p}\frac{y_e}{2}\right)\right],\tag{18}$$

$$\bar{u}_i=-\frac{a}{\sqrt{p}}\mathrm{th}\sqrt{p}\frac{y_e}{2}\left[1-\exp\left(-\frac{\sqrt{p}}{\varkappa}\xi\,\mathrm{cth}\sqrt{p}\frac{y_e}{2}\right)\right].\tag{19}$$

When changing from (18) and (19) to the originals we will take into account the fact that for fairly large times

$$\mathrm{ctg}\sqrt{p}\frac{y_e}{2}\approx\frac{2}{\sqrt{p}\,y_e}+\frac{1}{6}\sqrt{p}\,y_e$$

and the exponent in (18) and (19) takes the form

$$\exp\left(-\frac{\sqrt{p}}{\varkappa}\xi\,\mathrm{cth}\sqrt{p}\frac{y_e}{2}\right)\approx\exp\left[-(2\xi/\varkappa y_e)-(p\xi y_e/6\varkappa)\right].$$

Then, instead of (18) and (19) we have

$$\bar{u}_e=-\bar{u}_i=\frac{a}{\sqrt{p}}\mathrm{th}\sqrt{p}\frac{y_e}{2}-\frac{a}{\sqrt{p}}\exp\left(-\frac{2\xi}{\varkappa y_e}\right)\exp\left(-\frac{p\xi y_e}{6\varkappa}\right)\mathrm{th}\sqrt{p}\frac{y_e}{2}.$$

Changing to the originals and confining ourselves to the first terms of the series we obtain

$$u_e = u_i = \frac{ay_e}{2}\left[1-\exp\left(-\frac{2\xi}{\varkappa y_e}\right)\right]\left[1-\frac{8}{\pi^2}\exp(-\pi^2\theta/y_e^2)\right]. \tag{20}$$

Counter Flow (Fig. 1b). Using the first of conditions (10) and condition (11) we obtain the following two equations for $A(\xi)$ and $B(\xi)$:

$$\varkappa e^{\sqrt{p}\,y_e}A'(\xi)+\sqrt{p}\,e^{\sqrt{p}\,y_e}A(\xi)+\varkappa e^{-\sqrt{p}\,y_e}B'(\xi)-\sqrt{p}\,e^{-\sqrt{p}\,y_e}B(\xi)=a,$$

$$\varkappa A'(\xi)+\sqrt{p}\,A(\xi)+\varkappa B'(\xi)-\sqrt{p}\,B(\xi)=a.$$

The solution of this system gives

$$A(\xi)=C_1e^{-\frac{\sqrt{p}}{\varkappa}\xi}+\frac{a}{2\sqrt{p}}\frac{\exp\left(-\sqrt{p}\,\frac{y_e}{2}\right)}{\operatorname{ch}\sqrt{p}\,\frac{y_e}{2}},$$

$$B(\xi)=C_2e^{\frac{\sqrt{p}}{\varkappa}\xi}-\frac{a}{2\sqrt{p}}\frac{\exp\left(\sqrt{p}\,\frac{y_e}{2}\right)}{\operatorname{ch}\sqrt{p}\,\frac{y_e}{2}}.$$

Substituting these quantities into (14) we obtain

$$\bar{u}=\left[C_1e^{-\frac{\sqrt{p}}{\varkappa}\xi}+\frac{a}{2\sqrt{p}}\frac{\exp\left(-\sqrt{p}\,\frac{y_e}{2}\right)}{\operatorname{ch}\sqrt{p}\,\frac{y_e}{2}}\right]e^{\sqrt{p}\,y}+\left[C_2e^{\frac{\sqrt{p}}{\varkappa}\xi}-\frac{a}{2\sqrt{p}}\frac{\exp\left(\sqrt{p}\,\frac{y_e}{2}\right)}{\operatorname{ch}\sqrt{p}\,\frac{y_e}{2}}\right]e^{-\sqrt{p}\,y}. \tag{21}$$

From (21), assuming that $y = y_e$ and $y = 0$, we obtain the representations of the shift in concentration in the upper and lower channels

$$\bar{u}_e=C_1\exp\left[-\sqrt{p}\left(\frac{\xi}{\varkappa}-y\right)\right]+C_2\exp\left[\sqrt{p}\left(\frac{\xi}{\varkappa}-y\right)\right]-\frac{a}{\sqrt{p}}\frac{\operatorname{sh}\sqrt{p}\left(\frac{y_e}{2}-y\right)}{\operatorname{ch}\sqrt{p}\,\frac{y_e}{2}}, \tag{22}$$

$$\bar{u}_i=C_1\exp\left(-\sqrt{p}\,\frac{\xi}{\varkappa}\right)+C_2\exp\left(\sqrt{p}\,\frac{\xi}{\varkappa}\right)-\frac{a}{\sqrt{p}}\operatorname{th}\sqrt{p}\,\frac{y_e}{2}. \tag{23}$$

To obtain C_1 and C_2 we will assume that in the upper channel at $\xi = 0$ liquid enters with a concentration c_0, and enters with the same concentration in the lower channel at the opposite end ($\xi = 1$), i.e.,

$$\bar{u}_e|_{\xi=0}=0,\quad \bar{u}_i|_{\xi=1}=0. \tag{24}$$

From these conditions and (22) and (23) we obtain

$$C_1=\frac{a}{2\sqrt{p}}\frac{\exp\left[-\frac{\sqrt{p}}{2}\left(y_e-\frac{1}{\varkappa}\right)\right]}{\operatorname{sh}\frac{\sqrt{p}}{2}\left(y_e+\frac{1}{\varkappa}\right)}\operatorname{th}\sqrt{p}\,\frac{y_e}{2},$$

$$C_2=\frac{a}{2\sqrt{p}}\frac{\exp\left[\frac{\sqrt{p}}{2}\left(y_e-\frac{1}{\varkappa}\right)\right]}{\operatorname{sh}\frac{\sqrt{p}}{2}\left(y_e+\frac{1}{\varkappa}\right)}\operatorname{th}\sqrt{p}\,\frac{y_e}{2}.$$

Substituting these quantities into (22) and (23) we obtain, after appropriate reduction, with $\xi = 1$ and $\xi = 0$

$$\bar{u}_{e\text{к}}=\frac{2a}{\sqrt{p}}\frac{\operatorname{sh}\sqrt{p}\,\frac{y_e}{2}\operatorname{sh}\frac{\sqrt{p}}{2\varkappa}}{\operatorname{sh}\frac{\sqrt{p}}{2}\left(y_e+\frac{1}{\varkappa}\right)},$$

$$\bar{u}_{i\text{K}} = -\frac{2a}{\sqrt{p}} \frac{\operatorname{sh}\sqrt{p}\,\frac{y_e}{2}\,\operatorname{sh}\frac{\sqrt{p}}{2\varkappa}}{\operatorname{sh}\frac{\sqrt{p}}{2}\left(y_e + \frac{1}{\varkappa}\right)}, \tag{25}$$

where the subscript "K" corresponds to values of $\bar{u}$ at the exit of the apparatus.

Using tables of integral transformations, we can obtain the originals of (25)

$$u_{e\text{K}} = -u_{i\text{K}} = 2a\,\frac{\varkappa y_e + 1}{\pi^2\varkappa} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin^2 \frac{n\pi}{\varkappa y_e + 1}\left\{1 - \exp\left[-\frac{4\pi^2 n^2 \varkappa^2 \theta}{(\varkappa y_e + 1)^2}\right]\right\}.$$

Using the fact that

$$\frac{\varkappa y_e + 1}{\varkappa \pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin^2 \frac{n\pi}{\varkappa y_e + 1} = \frac{y_e}{2(\varkappa y_e + 1)},$$

we finally obtain

$$u_{e\text{K}} = -u_{i\text{K}} = \frac{a y_e}{\varkappa y_e + 1} - 2a\,\frac{\varkappa y_e + 1}{\pi^2\varkappa} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin^2 \frac{n\pi}{\varkappa y_e + 1} \exp\left[-\frac{4\pi^2 n^2 \varkappa^2 \theta}{(\varkappa y_e + 1)^2}\right]. \tag{26}$$

Since the approximation of the nonlinear term in (6) $c(1-c) = \approx 1/4$ which we assumed holds approximately in the range of values $0.3 < c < 0.7$, in the steady state the maximum permissible shift in concentration for which this approximation describes the process with satisfactory accuracy is $(c_{eK} - c_0)_{max} = (c_0 - c_{iK})_{max} = 0.2$. In addition, for this maximum value the logarithm of the degree of separation $\ln q \equiv y_e = \ln [c_{eK}/(1 - c_{eK})/c_{iK}/(1 - c_{iK})] = 1.7$. Taking these factors into account in the steady state we will have form (26)

$$0.2 > \frac{1.7}{4(1.7\varkappa + 1)} \quad \text{or} \quad \varkappa > 0.665. \tag{27}$$

Hence, (27) is the condition defining the limits of applicability of (26). It can be seen from (26) also that $u_{eK} = u_{iK} = 0$ as $\varkappa \to \infty$ or $y_e \to 0$, i.e., in this case separation will not occur.

The steady state will be achieved in practice when the exponent in the first term of series (26) is greater than four. This leads to the following estimate of the transient time:

$$\theta \geqslant \left(\frac{\varkappa y_e + 1}{\pi\varkappa}\right)^2. \tag{28}$$

The Closed Scheme. We will now consider the scheme shown in Fig. 1c. Its distinguishing feature is the fact that the liquid moves in a closed circuit in the channels. In this case, to obtain the constants in (22) and (23) we must use the condition for the concentrations at similar ends of the upper and lower channels to be the same, i.e.,

$$(c_e - c_i)_{\xi=0} = 0, \; (c_e - c_i)_{\xi=1} = 0. \tag{29}$$

Using these conditions we obtain

$$C_1 = -\frac{a}{2\sqrt{p}} \frac{\exp\left[\frac{\sqrt{p}}{2}\left(\frac{1}{\varkappa} - y_e\right)\right]}{\operatorname{ch}\frac{\sqrt{p}}{2\varkappa}\operatorname{ch}\sqrt{p}\,\frac{y_e}{2}},$$

$$C_2 = \frac{a}{2\sqrt{p}} \frac{\exp\left[-\frac{\sqrt{p}}{2}\left(\frac{1}{\varkappa} - y_e\right)\right]}{\operatorname{ch}\frac{\sqrt{p}}{2\varkappa}\operatorname{ch}\sqrt{p}\,\frac{y_e}{2}}.$$

Substituting this into (21) we obtain

$$\bar{u} = -\frac{a}{\sqrt{p}} \frac{\operatorname{sh}\left[\sqrt{p}\left(\frac{1}{2\varkappa} - \frac{\xi}{\varkappa} - \frac{y_e}{2} + y\right)\right]}{\operatorname{ch}\frac{\sqrt{p}}{2\varkappa}\operatorname{ch}\sqrt{p}\,\frac{y_e}{2}} - \frac{a}{\sqrt{p}} \frac{\operatorname{sh}\left[\sqrt{p}\left(\frac{y_e}{2} - y\right)\right]}{\operatorname{ch}\sqrt{p}\,\frac{y_e}{2}}. \tag{30}$$

Equation (30) also satisfies the integral condition

$$\frac{1}{y_e}\int_0^1\int_0^{y_e}\bar{u}\,dy\,d\xi=0,$$

which follows from the conditions for this system to be closed. The transform for the instantaneous shift in concentration in the upper and lower channels, according to (30), takes the form

$$\bar{u}_{e,i}=\pm\frac{a}{\sqrt{p}}\,\mathrm{th}\sqrt{p}\,\frac{y_e}{2}+\frac{a}{\sqrt{p}}\left\{\frac{\mathrm{sh}\,[\sqrt{p}\,(2\xi-1)/2\varkappa]}{\mathrm{ch}\,(\sqrt{p}/2\varkappa)}\mp\frac{\mathrm{ch}\,[\sqrt{p}\,(2\xi-1)/2\varkappa]}{\mathrm{ch}\,(\sqrt{p}/2\varkappa)}\,\mathrm{th}\left(\sqrt{p}\,\frac{y_e}{2}\right)\right\},$$

where the plus and minus signs relate to the upper and lower channels, respectively. We change back to the originals for the first two terms using expansion theorems, and the expansion and convolution theorems for the third term. As a result we obtain

$$u_{e,i}\equiv c_{e,i}-c_0=a\,\frac{2\xi-1}{2\varkappa}-$$

$$-\frac{4a}{\pi^2\varkappa}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2n-1)^2}\exp\left[-(2n-1)^2\pi^2\varkappa^2\theta\right]\sin\left[(2n-1)\frac{\pi}{2}(2\xi-1)\right]\pm$$

$$\pm\frac{2ay_e}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\exp\left[-(2n-1)^2\pi^2\varkappa^2\theta\right]\cos\left[(2n-1)\frac{\pi}{2}(2\xi-1)\right]\mp\frac{16ay_e}{\pi^3}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}(-1)^{n-1}\times$$

$$\times\frac{(2n-1)^2\varkappa^2y_e^2\exp\left[-(2n-1)^2\pi^2\varkappa^2\theta\right]-(2k-1)^2\exp\left[-(2k-1)^2\pi^2\theta/y_e^2\right]}{(2n-1)(2k-1)^2\left[(2n-1)^2\varkappa^2y_e^2-(2k-1)^2\right]}\cos\left[(2n-1)\frac{\pi}{2}(2\xi-1)\right]. \tag{31}$$

It is easy to show that when $\theta = 0$ the right side of (31) vanishes, i.e., condition (13) is satisfied.

Analysis of solution (31) enables us to draw the following conclusions.

1. In the steady state the instantaneous shift in concentration is a linear function of the length of the channel ξ while the difference in concentration between the ends of the apparatus

$$c|_{\xi=1}-c|_{\xi=0}=\frac{a}{\varkappa}. \tag{32}$$

It is noteworthy that this difference is independent of the height of the column, which obviously only holds in the limits of the approximation used. Considering the above, and (26) we obtain from (32)

$$0.4>\frac{a}{\varkappa}\quad\text{or}\quad\varkappa>0.625. \tag{33}$$

2. In any cross section of the channel in the steady state $c_e = c_i$, which should correspond to the absence of a concentration gradient along the height. This can easily be shown by differentiating (30) with respect to y and putting $p = 0$ (the steady state). The absence of a gradient, as follows from (2), leads to constancy of the transfer over the height of the column and, consequently, to the fact that the degree of separation is independent of y_e.

3. At the ends of the apparatus ($\xi = 0$, $\xi = 1$) the shift in concentration continues to be independent of the height of the column during the transient, whereas in the intermediate sections the shift in concentration depends on y_e.

4. The steady state is reached after a dimensionless time

$$\theta\geqslant\frac{4}{\pi^2\varkappa^2},$$

if $y_e \leq 1/\varkappa$, and after a time

$$\theta\geqslant\frac{4y_e^2}{\pi^2}, \tag{34}$$

if $y_e \gtrsim 1/\varkappa$. Note that (34) determines the transient time in a column closed at both ends and, consequently, by an appropriate choice of the rate of circulation ($\varkappa$) of the apparatus operating in the scheme illustrated in Fig. 1c, the same equilibrium concentration can be achieved in a much shorter time.

NOTATION

σ, mass flow rate of the liquid through the channel in unit time; D, binary diffusion coefficient; α, thermal diffusion constant; ρ, density; β, volume expansion coefficients; δ, gap, i.e., the distance between the two constant-temperature surfaces; $\Delta T = T_2 - T_1$; T_2, T_1, temperatures of the heated and cooled surfaces $T = (T_1 + T_2)/2$; η, dynamic viscosity; B, length; L, height of the apparatus; z, vertical coordinate; x, longitudinal coordinate; $K = g^2\rho^3\beta^2\delta^7(\Delta T)^2 B/9!\eta^2 D$; c, mass concentration; τ^*, transfer of the purposeful component in the vertical direction in units of mass in unit time per unit length of the apparatus; $m^* = \rho\delta$; t, time; f, area of transverse cross section of the channel; w, rate of flow of the liquid in the channel; $y_e = HL/K$; ω, θ, ξ, $\varkappa$, see Eqs. (8); u, see Eq. (12); and p, an operator. Subscripts: e, upper channel; i, lower channel; 0, initial value, and K, end of the apparatus.

LITERATURE CITED

1. G. D. Rabinovich, "Theory of thermal diffusion separation using the Frazier scheme," Inzh.-Fiz. Zh., 31, No. 3 (1976).

STABILITY OF THE INTERPHASE SURFACE IN THE FREEZING OF MOIST GROUND

Yu. S. Daniélyan and P. A. Yanitskii

UDC 624.131.4/5:625.123

It is suggested that the formation of ice layers should be regarded as a consequence of a loss of stability of the motion of the freezing front. The kinetics of the freezing process is investigated and a stability criterion is obtained.

The freezing of moist ground is accompanied by the migration of moisture, i.e., by a redistribution of the initial moisture. Experiments show that in different grounds and for different modes of freezing the redistribution of the moisture leads to various textures of the frozen rocks. In particular, in certain cases ice layers are formed, and sometimes monotonic freezing, etc., occurs. There are different ways of describing the various aspects of this phenomenon. Examples are given in [1, 2]. Below we carry out a theoretical analysis based on a study of the stability of the process. The formation of ice layers is treated as a consequence of the loss of stability of the motion of the freezing front with respect to small perturbations.

The one-dimensional freezing problem, taking into account the migration of moisture, can be described in the following form [3]:

$$\frac{\partial T_1}{\partial t} = a_1 \frac{\partial^2 T_1}{\partial x^2}, \quad 0 < x < s(t),$$

$$\left.\begin{aligned} \frac{\partial T_2}{\partial t} &= a_2 \frac{\partial^2 T_2}{\partial x^2} \\ \frac{\partial W}{\partial t} &= K \frac{\partial^2 W}{\partial x^2} \end{aligned}\right\} \quad s(t) < x < L.$$

The initial conditions are

$$W(x, 0) = W_0, \quad T_2(x, 0) = T_н,$$

and the boundary conditions are

V. I. Muravlenko Giprotyumenneftegaz. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 39, No. 1, pp. 96-101, July, 1980. Original article submitted July 2, 1979.